

Letters

Comments on “Controller Synthesis of Fuzzy-Dynamic Systems Based on Piecewise Lyapunov Functions”

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Abstract—This comment tries to describe a theoretical mistake made in the aforementioned paper [G. Feng, *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 5, pp. 605–612, Oct. 2003] to formulate the inverse of matrices used to construct the piecewise-quadratic Lyapunov functions. Derivation of these inverse matrices is the most critical step toward transforming the design constraints into linear matrix inequalities (LMIs). Therefore, the erroneous formulation essentially affects the validity of the approach and final results. Unfortunately, it seems that there is no simple correction for this problem. However, some close alternative approaches are suggested by the same authors in their other works.

Index Terms—Fuzzy systems, piecewise-quadratic Lyapunov function, stabilization.

I. INTRODUCTION

PIECEWISE-QUADRATIC candidates of Lyapunov function, due to more flexibility with respect to common quadratic functions, have received much attention to stability analysis and stabilization of dynamical fuzzy systems. These piecewise functions are, in general, discontinuous across the cell (subspace) boundaries. Because of these discontinuities, some boundary conditions are required to be checked, in addition to intercell conditions, to guarantee the stability of the system. In order to avoid the difficulties caused by these boundary conditions, Feng [1] presents a controller-design technique, which is based on the continuous piecewise-quadratic functions of the form introduced in [2], and tries to derive the stabilizing design constraints, which are in the form of linear matrix inequalities (LMIs). The piecewise-quadratic function in [2] is parameterized using some constraint matrices, i.e., F_l , of full column-rank, to preserve the continuity of the function across the cell boundaries. However, this special type of parameterization makes some difficulties in transforming the closed-loop stability conditions into LMIs, which are mainly because of inability to formulate the inverse of the parameterized matrices. This matrix inversion has wrongly been formulated as [1, eq. (2.13)]. In this paper, we try to notify this theoretical mistake and show how it invalidates the main results of Feng [1].

Manuscript received April 17, 2009; accepted October 28, 2009. First published November 6, 2009; current version published February 5, 2010.

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Digital Object Identifier 10.1109/TFUZZ.2009.2036006

II. ERROR AND ITS CONSEQUENCES

As presented in [2], the piecewise-quadratic candidate of Lyapunov function and its special parameterization are written in the l th cell, i.e., \bar{S}_l , as follows:

$$V(x) = x^T P_l x, \quad x \in \bar{S}_l \quad (1)$$

$$P_l = F_l^T T F_l \quad (2)$$

where F_l is a constraint matrix of full column-rank. The special parameterization form (2) preserves the continuity of (1) across the cell boundaries. Now, consider the proof of [1, Th. 2.1]. A matrix P_l is formulated as follows [1, eq. (2.13)]:

$$P_l = (F_l^T F_l)^{-1} F_l^T T F_l (F_l^T F_l)^{-1} \quad (3)$$

whose inverse matrix is considered as [1, eq. (2.17)]

$$P_l^{-1} = F_l^T T^{-1} F_l. \quad (4)$$

Although (4) is in the form of (2) and preserves the continuity of the function [1, eq. (2.16)], it is not the inverse of (3). As stated before, F_l is a matrix of full column-rank, and in the approach of fuzzy-model-based control, which is taken in [1], it is always nonsquare (see the construction method of F_l in [2]). Therefore, if we form the matrices $P_l P_l^{-1}$ and $P_l^{-1} P_l$, we obviously find them to not be equal to the identity matrix. Also, we can simply check this inequality via a numerical example. Consider the results of the simulation example in [1, Sec. IV]. For the first subspace \bar{S}_1 , we have

$$P_1 P_1^{-1} = (P_1^{-1} P_1)^T = \begin{bmatrix} -7.3945 & -2.7356 & 8.4069 & 1.7149 \\ 13.7915 & 5.4943 & -13.812 & -2.817 \\ 2.1098 & 0.6875 & -1.1129 & -0.431 \\ -0.6380 & -0.2079 & 0.6389 & 1.1303 \end{bmatrix} \neq I$$

which clearly shows the invalidity of (3) as an inverse for (4), and *vice versa*.

Now, we show the critical consequences of this error. $P_l P_l^{-1} = I$ and $P_l^{-1} P_l = I$ are the key properties used in [1, Th. 2.1 and Lemma 3.1]. The inequality [1, eq. (2.21)] in the proof of [1, Th. 2.1] is obtained by multiplying its previous inequality on the left-hand side and right-hand side by P_l^{-1} . Also, the following inequality in the proof of [1, Lemma 3.1] is written based on the constraint [1, eq. (3.8)] by the same right-hand-side

and left-hand-side multiplication by $R_l = P_l^{-1}$

$$\begin{aligned} & \int_0^\infty [x^T (A_{cl}^T R_l + R_l A_{cl})x + v^T D_{cl}^T R_l x + x^T R_l D_{cl} v] dt \\ & < \int_0^\infty [x^T (-\gamma^{-2} R_l D_{cl} D_{cl}^T R_l - H_{cl}^T H_{cl})x \\ & \quad + v^T D_{cl}^T R_l x + x^T R_l D_{cl} v] dt. \end{aligned} \quad (5)$$

Both of these multiplications are correct because the constraints [1, eq. (2.14) and eq. (3.7)] ensure the existence of P_l^{-1} . However, the main problem arises in the construction of the candidate for Lyapunov function [1, eq. (2.16) and eq. (3.9)]. Since we showed that the inverse of (3) is not in the form of (4), the candidate for Lyapunov function is not necessarily continuous across the cell boundaries. This can also be seen via the numerical example of the previous section. If we consider a boundary point, e.g., $x_b = [0, 10, 0, 0]^T$ and calculate the resulting Lyapunov function at this boundary point, we come up with $V(x_b^-) = x_b^T P_1^{-1} x_b = 417.8176$ and $V(x_b^+) = x_b^T P_2^{-1} x_b = 414.2692$, which clearly implies the discontinuity of the Lyapunov function. Therefore, despite the main objective of [1], the boundary conditions are still required to ensure the stability of closed-loop and open-loop systems. Consequently, the constraints of [1, Th. 3.1], which are derived mainly based on [1, Lemma 3.1] and summarize the main contribution of [1], are not sufficient for stability of closed-loop system.

To find a formulation for the inverse of (2) is crucial to stability analysis of the closed-loop system. Actually, the inability to formulate the inverse of (2) is the bottleneck in transforming the closed-loop stability constraints into LMIs. In the proof of [1, Lemma 3.1], if we substitute A_{cl} in the first equality, we have

$$\begin{aligned} & \int_0^\infty \frac{d}{dt} (x^T R_l x) dt \\ & = \int_0^\infty \{x^T [K_l^T B_l^T R_l + R_l B_l K_l + \dots] + \dots\} dt. \end{aligned}$$

The term $K_l^T B_l^T R_l + R_l B_l K_l$ remains unchanged in all consequent steps toward derivation of the final constraints and prevents the final constraints to be transformed into LMIs. It is not very straightforward to solve this problem and find the closed-loop stability constraints in the form of LMIs.

III. SOME CORRECT ALTERNATIVES

The problem described in the previous section can be easily resolved for the stability analysis of the open-loop system. The constraints required to ensure the stability of the open-loop system can be found in [2] in the form of LMIs, where there is no need to introduce an inverse for (2). On the contrary, as stated in previous section, a crucial problem arises while dealing with the closed-loop system. Currently, some alternative approaches are available in the next works of Feng [1], e.g., in [3]–[6], in which the aforementioned theoretical mistake has been corrected by the cost of some minorities in the claimed contributions.

IV. CONCLUSION

This paper revealed a critical theoretical mistake in [1] and its consequences on the obtained results. It was shown that the final LMI-form constraints presented in [1, Th. 3.1] are insufficient to ensure the stability of the closed-loop fuzzy-dynamical system. Unfortunately, no simple correction could be carried out. In order to retrieve the primary goal of Feng [1], see the alternative approaches proposed in the next works of Feng [1].

REFERENCES

- [1] G. Feng, "Controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 5, pp. 605–612, Oct. 2003.
- [2] M. Johansson, A. Rantzer, and K. E. Årzén, "Piecewise quadratic stability of fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 6, pp. 713–722, Dec. 1999.
- [3] G. Feng, "Controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions and bilinear matrix inequalities," in *Proc. 12th IEEE Int. Conf. Fuzzy Syst.*, St. Louis, MO, May 2003, vol. 2, pp. 1327–1332.
- [4] G. Feng, "H-infinity controller design of fuzzy dynamic systems based on piecewise Lyapunov functions," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 34, no. 1, pp. 283–292, Feb. 2004.
- [5] G. Feng, C. L. Chen, D. Sun, and X. P. Guan, "H-infinity controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions and bilinear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 1, pp. 94–103, Feb. 2005.
- [6] B. Yang, D. Yu, G. Feng, and C. Chen, "Stabilization of a class of nonlinear continuous time systems by a fuzzy control approach," in *Inst. Electr. Eng. Proc. Control Theory Appl.*, Jul. 2006, vol. 153, no. 4, pp. 427–436.